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CONTINUOUS QUANTUM COMPUTATION

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14. ABSTRACT <p>The focus of this research was on developing quantum algorithms for continuous problems, complexity analysis of these algorithms, and their simulation and implementation. Continuous problems are a focus because much of physics, chemistry, and engineering depends on continuous mathematical formulations such as partial differential equations, path integration, approximation, and high-dimensional integration. New algorithms and quantum speedups were obtained for a number of important problems such as path integration, eigenvalues of Hermitian operators, Feynman-Kac path integration, high-dimensional approximation, and the Sturm-Liouville eigenvalue problem. The simulation and implementation part of the project included simulation of the quantum summation algorithm, implementation of the quantum Baker's map, NMR implementation of a quantum lattice gas, application of a Loschmidt echo, single spin measurement, and experiments in solid-state simulation.</p>					
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1.0 Introduction

Two of the major motivations for solving continuous problems on a quantum computer are:

- A. Many scientific problems have continuous mathematical formulas. Examples of such formulations are:
 - Path Integration
 - Feynman-Kac path integration
 - Schrodinger equation
- B. In their standard monograph, Nielsen and Chuang [1] state:
"Of particular interest is a decisive answer to the problem whether quantum computers are more powerful than classical computers." To answer this question one must know the classical and quantum computational complexities.

By computational complexity (complexity for brevity) is meant the *minimal* computational resources needed to solve a problem. Two of the most important resources for quantum computing are qubits and queries. Classical complexity has been extensively studied in information-based complexity [2], [3].

The classical complexity of many continuous problems is known. Therefore, when the complexity of these problems is obtained, the question as to whether quantum computers are more powerful than classical computers can be answered. Furthermore, it can be established how much more powerful. In contrast, the complexity of discrete problems is typically unknown; one has to settle for conjectures about the complexity hierarchy. For example, the classical complexity of integer factorization is unknown. It is believed that the classical complexity of integer factorization is super-polynomial and that, therefore, Shor's quantum algorithm for factorization is much faster, but there is no proof. The reason why one can establish the complexity of continuous problems but has to settle for conjectures about the complexity hierarchy of discrete problems is discussed in [4].

In this project, new algorithms and quantum speedups were obtained for a number of important problems such as path integration, eigenvalues of Hermitian operators, Feynman-Kac path integration, high dimensional approximation, and the Sturm-Liouville eigenvalue problem.

It would be desirable to implement the new quantum algorithms on quantum computers. Of course, at this time, only a few qubits are available. Nonetheless, a variety of experiments have been carried out using available quantum computers, or by simulation.

2.0 Algorithms and Complexity

2.1 Multivariate Approximation

The study of quantum complexity, tractability, and strong tractability for multivariate approximation was initiated. A space of functions important in many applications (technically this is called a Korobov space) was studied. To report on this work a couple of concepts have to be reviewed. A function space is "weighted" if certain variables are more important than others; the weights show the relative importance of the variables. That is, the functions are non-isotropic. In an unweighted space, all variables are equally important; the functions are isotropic.

A second major concept is that of strong tractability. A multivariate problem is strongly tractable if its complexity is independent of the problem's dimension; the complexity depends only on the desired error.

It is known that in the worst case setting on a classical computer the approximation problem is intractable (it suffers the "curse of dimensionality") for both unweighted and weighted Korobov spaces.

There is good news and bad news for quantum computing. The good news is that under a certain assumption on the weights this problem is strongly tractable on a quantum computer. The bad news is that it remains intractable for unweighted spaces even on a quantum computer.

See 2.2 for more results and a reference.

2.2 Multivariate Approximation Continued

If the sum of a certain power of the weights is finite, then multivariate approximation is strongly tractable in both quantum and randomized settings. In the quantum setting, an algorithm with error at most E that uses only about $d + \log(1/E)$ qubits, where d is the number of variables, was designed. Hence, there is only linear dependence on d and logarithmic dependence on $1/E$. For many practical values of d and E the number of qubits is thus quite modest.

The total cost of this algorithm is polynomial in $1/E$ and is roughly $(1/E)^{(1+r)}$ times smaller than the randomized complexity of the approximation problem. Here r is a positive parameter that depends on the weights and may be large. This means that the speed-up of quantum over classical computers may be much larger than quadratic. Quadratic speed-up has been established for a number of problems. Hence, multivariate approximation is probably the first example of an important continuous problem for which quantum computation can lead to larger than quadratic speed-up over the classical worst case [30].

2.3 Path Integration

Path integration on a quantum computer was studied. Let E be the desired error. The main conclusions are: Path integration on a quantum computer is tractable, and can be solved roughly $1/E$ times faster than on a classical computer using randomization, and exponentially faster than on a classical computer with a worst case assurance. The number of quantum queries is the square root of the number of function values needed on a classical computer using randomization. The number of qubits is a low degree polynomial in $1/E$ [10].

2.4 Error Bounds of Quantum Summation

Error bounds on the quantum summation (QS) algorithm for other settings were studied. The first such setting to be studied was the average, rather than worst, behavior of the QS algorithm. For this setting a distribution was assumed. One natural assumption is that all inputs are equiprobable. Then there was a surprising result. If M , the number of quantum queries, is divisible by 4, then the upper bound is of order $\min(1/M, q)$ where $q = 1/\sqrt{N}$. Of course, it can always be arranged that M is divisible by 4, so this result holds without loss of generality. If, on the other hand, it is assumed that all outputs (rather than inputs) are equiprobable, then the upper bound is of order $1/M$ [28] [16].

2.5 Eigenvalue of Hermitian Operator

Quantum mechanical problems offer great potential for quantum computers to achieve large speedups over classical machines. An important problem of this kind is approximation of an eigenvalue of a Hermitian operator. In a recent paper Abrams and Lloyd [5] presented a quantum algorithm for doing this. Their algorithm is exponentially faster than the best classical algorithm, but requires a good approximation of an eigenvector as input. It has been shown how to obtain such an approximation efficiently which is guaranteed to be good [17].

2.6 Universal Quantum Control

The quantum state-space H encoding information decomposes into irreducible sectors and subsystems associated to the group of available evolutions. If this group coincides with the unitary part of the group-algebra of some group K then universal control is achievable over K -irreducible components of H . This general strategy is applied to different kinds of bosonic systems [37].

2.7 Compiling Quantum Circuits

No matter what technology will ultimately be used to implement quantum computers, the quantum circuit is most likely to remain the primary model for quantum computation. A general purpose quantum compiler will require both technology-independent and technology-dependent optimization techniques. Until a scalable quantum computer technology emerges, development is restricted to machine-independent techniques. New algorithms for compiling arbitrary unitary matrices of order 2^m into efficient circuits of $(m-1)$ -controlled single-qubit and $(m-1)$ -controlled-NOT gates have been developed. The Palindromic Optimization Algorithm significantly reduces the number of gates generated by the conventional method [14].

2.8 Complexity of Quantum Summation

Summation on a quantum computer is a basic part of the toolbox for computing path integrals, high-dimensional integrals, and high-dimensional approximations. A particular algorithm, the quantum summation (QS) algorithm of Brassard, Hoyer, Mosca and Tapp, which approximates the arithmetic mean of a Boolean function defined on N elements, is known to be optimal in the usual setting studied in quantum computation, the worst-probabilistic setting. The QS algorithm has now been analyzed in the worst-average setting; that is worst with respect to all inputs and average with respect to all outcomes. Assume M queries. Using a number of repetitions which is independent of M , the upper bound on the QS algorithm is of order $1/M$. Since the lower bound on the worst-average error of any algorithm is $1/M$, the QS algorithm with repetitions is optimal in this setting.

Since the complexity of summation on a quantum computer is of order $1/M$ in both the worst-probabilistic and worst-average setting and the QS algorithm is optimal in both settings, what is the significance of the result? In the worst-probabilistic setting (which is the standard setting of quantum computation) a good answer is guaranteed with probability p , with probability $1-p$, the answer can be arbitrarily poor. In the worst-average setting, a good answer is guaranteed averaged over all outcomes [35].

2.9 Query Complexity

Query complexity measures the amount of information an algorithm needs about a problem to compute a solution. On a quantum computer there are different realizations of a query and it is shown that these are not equivalent.

It was shown that a bit query can always approximate a phase query with just two queries, while there exist problems for which the number of phase queries which are necessary to approximate a bit query must grow exponentially with the precision of the bit query [32].

2.10 LDOS on a Quantum Computer

An efficient quantum algorithm for estimating the local density of states (LDOS) on a quantum computer has been obtained. The LDOS describes the redistribution of energy levels of a quantum system under the influence of a perturbation. Sometimes known as the "strength function" from nuclear spectroscopy experiments, the shape of the LDOS is directly related to the survival probability of unperturbed eigenstates. For quantum systems that can be simulated efficiently on a quantum computer, the LDOS estimation algorithm enables an exponential speed-up over direct classical computation [34].

2.11 Feynman-Kac Path Integral on a Quantum Computer

Many phenomena in quantum chemistry or quantum physics require the calculation of the Feynman-Kac path integral. Furthermore, the Feynman-Kac path integral formula can be used to solve the heat equation in d space variables. Algorithms and complexity for Feynman-Kac integration in three settings: classical worst case deterministic, classical randomized, and quantum were studied. In the classical randomized and quantum settings the curse of dimensionality is broken. In the classical randomized setting the complexity of computing an answer with error tolerance E is roughly $(1/E)^2$, whereas in the quantum setting it is roughly $1/E$. Thus the quantum complexity enjoys exponential speedup over the classical worst case and polynomial speedup over the classical randomized. The quantum algorithm uses roughly $(1/E)$ bit queries and $\log(1/E)$ qubits. The logarithmic qubit dependence is particularly gratifying since qubits will be a scarce resource for the foreseeable future [38].

2.12 Adiabatic Quantum Computation

Adiabatic quantum computation is a technique in which one encodes the answer to a hard problem, such as an NP-complete problem, in the ground state of a simple Hamiltonian, then adiabatically deforming the Hamiltonian in to the one whose ground state encodes the answer to the problem. One can also think of adiabatic quantum computation as an algorithm, in which one uses a quantum computer to simulate the adiabatic dynamics. The key issue in adiabatic quantum computation is the size of the gap between the ground state and the first excited state. If this gap does not become exponentially small during the course of the adiabatic process, then adiabatic quantum computation is efficient. It was shown that ground state quantum computation has a gap that is only polynomially small (it goes as one over the length of the computation squared). Accordingly, ground state quantum computation is efficient. In addition, ground state quantum computation is immune to a variety of errors: in particular, because the system remains in the ground state throughout, ground state quantum computation is resistant to the most common type of decoherence, decoherence in the energy eigenbases. The possibility of constructing such adiabatic quantum computers using superconducting systems is being investigated [42] [43].

2.13 For Which Quantum Queries Can NP-Complete Problems Be Solved

For which quantum queries can NP-complete problems be solved? The power of quantum computation depends on the power of the permitted queries. There has been much interest on whether NP-complete problems are solvable in polynomial time on a quantum computer. The goal is to investigate the use of more powerful queries than have been used in the past. The queries under consideration are those used in the phase estimation algorithm. These are referred to as power queries because they involve powers of a unitary matrix. For some U , the j -th power can be implemented at polylog cost in j . This is the case for Shor's algorithm. How many power queries are needed to solve NP-complete problems? Here is a possible attack. Perform a series of reductions as follows:

- reduce SAT to quantum summation
- reduce quantum summation to integration
- reduce integration to the Sturm-Liouville eigenvalue problem (SLE)
- solve SLE

Note that a discrete problem, SAT, has been reduced to the continuous problem SLE. Preliminary research indicates that SLE can be solved by the phase estimation algorithm using power queries for the matrix $U = \exp(i(A+B))$. Here A is a tridiagonal matrix related to the quantum Fourier transform and B is diagonal. A is fixed; only B depends on the Boolean function specifying SAT. The intent is to show that the number of power queries for SAT will be polynomial in the number of variables of SAT. Even if the number of power queries is polynomial, the cost of solving SAT depends on the cost of power queries. To solve NP-complete problems in polynomial time, power queries in polylog time by digital or analog means will need to be computed. For which diagonal matrices is this possible? In particular, does this hold for B which encodes SAT? [47]

2.14 Adiabatic Quantum Computation and Path Integration

Significant progress was made relating adiabatic quantum computation to path integration. It was shown that replica methods for spin glasses can be applied to calculate gaps for adiabatic quantum computing. The results suggest that adiabatic quantum computation should be able to solve average case NP-complete problems such as random 3-SAT or MAX Clique. This result suggests that adiabatic quantum computing is a powerful technique for solving a wide variety of hard optimization problems [15].

2.15 Randomized Quantum Query Setting and Exponential Qubit Speedup

A critical resource for the foreseeable future is the number of qubits. Exponential improvement in the qubit complexity for important problems has been obtained. This is achieved by introducing a new setting for quantum computation in which quantum queries are randomized. This setting will be referred to as the randomized quantum query setting (RQQ). This is the quantum counterpart of the randomized classical (Monte Carlo) setting. Note that the standard quantum setting uses deterministic queries and guarantees solutions with a certain probability. Randomness only occurs during measurement. In the standard setting it was shown earlier that a path integral could be computed with $1/E$ qubits and $1/E$ queries. Here E is the error threshold. In the RQQ setting, path integration can be done with $\log(1/E)$ qubits and $1/E$ queries; there is exponential improvement in the number of qubits. These results are best possible; the problems' complexities are known [55] [4].

3.0 Simulation and Implementation

3.1 Implementation of Integration in Nuclear Magnetic Resonance (NMR) Quantum Information Processing (QIP)

An experimental protocol for implementing integration on an NMR quantum information processor was developed. In particular:

- A. investigated protocols for performing summation and integration on 3 and 5 qubit NMR quantum information processor,
- B. devised robust pulse sequences to accomplish sequence of, quantum logic operations without ex post facto fine tuning of the sequences,
- C. programmed a NMR simulator to investigate the properties of different pulse sequences,
- D. analyzed the effects of noise and decoherence on integration algorithms.

3.2 Simulation of the Quantum Summation Algorithm

The quantum algorithm of Brassard, Hoyer, Mosca and Tapp that is based on Grover's quantum search algorithm was simulated. There are two versions of this simulation. The first one simulates all intermediate steps of the quantum algorithm whereas the second one simulates only quantum results before measurement. Let N be the number of terms in the sum, the cost of the first simulation is roughly $N(1/E)\log(1/E)$, whereas the cost of the second simulation is much cheaper and is roughly equal to $N+(1/E)\log(1/E)$.

3.3 Implementation of the Quantum Baker's Map

An experimental implementation of the quantum Baker's map via a three qubit NMR quantum information processor has been completed. The experiments tested the sensitivity of the quantum chaotic map to perturbations. These experiments can be used to investigate existing theoretical predictions for quantum chaotic dynamics [7].

3.4 NMR Implementation of a Quantum Lattice Gas Algorithm

An ensemble nuclear magnetic resonance implementation of a quantum lattice gas algorithm for the diffusion equation was studied. The algorithm employs an array of quantum information processors sharing classical information, a novel architecture referred to as a type-II quantum computer. This concrete implementation provides a test example from which to probe the strengths and limitations of this new computation paradigm. The NMR experiment consists of encoding a mass density onto an array of 16 two-qubit quantum information processors and then following the computation through 7 time steps of the algorithm. The results show good agreement with the analytic solution for diffusive dynamics. Numerical simulations of the NMR implementation are described. The simulations aid in determining sources of experimental errors and they help define the limits of the implementation [13].

3.5 Robust Control of Quantum Information

Incoherent errors can be described, on average, by completely positive superoperators, but can nevertheless be corrected by the application of a locally unitary operation that "refocuses" them. They are due to reproducible spatial or temporal variations in the system's Hamiltonian, so that information on the variations is encoded in the system's spatiotemporal state and can be used to correct them. Liquid-state nuclear magnetic resonance is used to demonstrate that such refocusing effects can be built directly into the control fields, where the incoherence arises from spatial inhomogeneities in the quantizing static magnetic field as well as the radio-frequency control fields themselves [21].

3.6 Compensation of Decoherence with Bang-Bang Control

With the growing efforts in isolating solid-state qubits from external decoherence sources, the origins of noise inherent to the material start to play a relevant role. The random walk of the qubit state on the Bloch sphere with and without bang-bang compensation by means of the stochastic Schroedinger equation was simulated. The analysis gives the effect of bang-bang control on the entire distribution. Bang-bang control works as a high-pass filter on the spectrum of noise sources. This indicates how the influence of $1/f$ -noise ubiquitous to the solid state world can be reduced [18].

3.7 Quantum Pseudo-Random Generators

Random numbers play a key role in a variety of algorithms, notably Monte Carlo algorithms. Similarly, random states and random unitary transformations are useful in quantum algorithms. But true random numbers and random unitary transformations are hard to construct. Consequently, classical Monte Carlo algorithms typically use pseudo-random "seeds" into long sequences of numbers that are "random enough". The quantum analog of pseudo-random number generators, small quantum circuits that perform pseudo-random unitary transformations, were constructed. Quantum pseudo-random transformation generators were investigated both theoretically and experimentally. It was shown that pseudo-random transformations were "random enough" for the purposes of investigating quantum chaos and generating entanglement. Simple pseudo-random transformations on room-temperature NMR quantum information processors were implemented.

3.8 Single Spin Measurement

A clean model for single spin measurement has been provided and the essential elements have been experimentally demonstrated on a simplified system. The model can be used to explore the challenges in applying this approach to a single spin, and gives the experimentalists a goal for coherent control. It clearly shows that arrays of spins can be engineered to allow quantum information processing on pure, atomic systems (such as Si-29 patterned in Si-28). The significance of this effort is that it provides a quantum information based approach to measuring the state of a single spin. This will eventually enable moving from an ensemble-based approach to quantum computing, to a single qubit, pure state approach [45] [46].

3.9 Experiments in Solid-State Simulation

Additional experiments in solid-state quantum simulation produced results that show that coherent processes such as spin diffusion in solids can be highly enhanced by the presence of entanglement. In addition, decoherence rates for highly entangled states in a variety of solid-state systems were investigated. Highly entangled states decay significantly more slowly than predicted, suggesting the existence of previously unsuspected physical effects [40].

4.0 Summary

One of the goals of the project was to study the quantum speedup of important mathematical models. In every case that was studied, there was significant quantum speedup. For example, for path integration (see Section 2.3) there is exponential speedup over the classical worst case and polynomial speedup (that is, Grover-type speedup) over the classical randomized case. The important problem of approximating an eigenvalue of a Hermitian operator enjoys exponential quantum speedup (see Section 2.5). Multivariate approximation may enjoy polynomial quantum speedup of arbitrarily high degree (see Section 2.2).

Two results with considerable promise were achieved recently:

I. It has been shown by a series of reductions that SAT can be reduced to a one dimensional Sturm-Liouville eigenvalue problem (see Section 2.13). This is a counter-intuitive result relating a famous discrete problem to a one-dimensional continuous one. Does this imply that NP-complete problems can be solved on quantum computers? There's no question about the reductions; they're solid. However, to solve NP-complete problems in polynomial time, power queries would have to be computed in polylog time. It is an open question whether this can be done.

II. A fundamental proposition of quantum computing should be:

ANY COMPUTATION THAT CAN BE PERFORMED ON A CLASSICAL
COMPUTER CAN BE PERFORMED ON A QUANTUM COMPUTER.

A problem was identified that can be solved on a classical computer using the Monte Carlo algorithm but cannot be solved on a quantum computer in the standard setting. In the standard setting evolution is deterministic; only the results of a measurement are probabilistic. A new setting has been introduced in which quantum queries are randomized (see Section 2.15). In this setting the problem mentioned above can be solved.

It must be stressed that this desirable result is due to the new setting and says nothing fundamental about the nature of quantum computing. When path integration was studied in this setting an exponential reduction in the qubit complexity was obtained. This is particularly important since the number of qubits is a critical resource for the foreseeable future. This new setting merits further study.

Among the major accomplishments in the experimental portion of the project were the following:

- A. Developed and experimentally implemented algorithms for path integration and quantum simulation on both small-scale liquid state NMR quantum information processors, and large scale solid state NMR quantum information processors.
- B. Devised new designs and architectures for adiabatic quantum computation.
- C. Devised theoretical analysis of the role of entanglement in regular and chaotic quantum dynamics and performed experimental tests on those analyses.
- D. Designed and implemented quantum random transformation generators.
- E. Developed the essential pieces for solid state approaches to QIP.
- F. Showed the necessary control in large Hilbert spaces (up to ~60 spin coherences) and showed selective spin transfer between two dipolar coupled spins without leakage to other adjacent spins.

A number of lessons were learned in the experimental work. It was harder than expected to develop experimental continuous and hybrid quantum systems. It was easier to harness solid-state NMR quantum information processors than expected as they turned out to be flexible enough to investigate the kind of entangled complex dynamics of interest.

5.0 List of Presentations

S. Lloyd, Veridian Colloquium, Ann Arbor, MI, July 2001
S. Lloyd, NRO, Chantilly, VA, August 2001
H. Wozniakowski, The Australian National University, Canberra, Australia, August 2001
H. Wozniakowski, DARPA, Columbia University, NY, NY, September 2001
J.F. Traub, Los Alamos National Laboratory, Santa Fe, NM, October 2001
S. Lloyd, Solvay Conference, Delphi, Greece, November 2001
H. Wozniakowski, QuIST, Dallas, TX, November 2001
H. Wozniakowski, Oberwolfach, Germany, November 2001
S. Lloyd, Department of Applied Mathematics, Cambridge, England, December 2001
S. Lloyd, Hewlett Packard, Palo Alto, CA, February 2002
H. Wozniakowski, Courant Institute, February 2002
H. Wozniakowski, Department of Physics, Columbia University, NYC, NY, March 2002
H. Wozniakowski, DARPA, MIT, April 2002
S. Lloyd, Hewlett Packard, Palo Alto, CA, July 2002
H. Wozniakowski, FoCM, Minneapolis, MN, August 2002
J.F. Traub and H. Wozniakowski, Los Alamos National Laboratory, Santa Fe, NM, August 2002
S. Lloyd and H. Wozniakowski, QuIST, Cambridge, MA, September 2002
J.F. Traub, Carnegie Mellon University, Pittsburgh, PA, September 2002
J.F. Traub and H. Wozniakowski, Hanscomb Air Force Research Laboratory, September 2002
S. Lloyd, Quantum Control Workshop, MIT, October 2002
S. Lloyd, BES-AMOS Meeting, VA, October 2002
J.F. Traub, Peking University, Beijing, PRC, October 2002
J.F. Traub, Fudan University, Shanghai, PRC, October 2002
J.F. Traub, University of Tokyo, Japan, October 2002
S. Lloyd, Physics Colloquium, Washington University, St. Louis, MO., November, 2002
S. Lloyd, Physics Colloquium, University of Michigan, November 2002
H. Wozniakowski, MCQMC 2002, Singapore, November 2002
S. Lloyd, Lawrence Livermore National Laboratory, LLNL, CA., January 2003
J.F. Traub, American Mathematical Society, Baltimore, MD., January 2003
H. Wozniakowski, University of Goettingen, Germany, February 2003
J.F. Traub, DARPA, MIT, March 2003
S. Lloyd, Delft Technical Institute, Delft, Netherlands, May 2003
D. Cory, Harper's Ferry, June 2003
D. Cory, Canadian Institute for Advanced Research, Baniff, Canada, June 2003

H. Wozniakowski, ICIAM 2003, Sydney, Australia, July 2003
 S. Lloyd, EQIS, Kyoto, Japan, September 2003
 S. Lloyd, Tokyo Institute of Technology, November 2003
 J.F. Traub, Santa Fe Institute, Santa Fe, NM, November 2003
 H. Wozniakowski, Australian Research Council, Melbourne, Australia,
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 S. Lloyd, NTT Quantum Information Conference, Amsterdam,
 Netherlands, December 2003
 K. Svore, ERATO, Kyoto, Japan, December 2003
 J.F. Traub, University of New South Wales, Sydney, Australia,
 December 2003
 S. Lloyd and J.F. Traub, Gordon Conference, Ventura, CA, February
 2004
 S. Lloyd, RIKEN, Japan, February 2004
 J.F. Traub, Perimeter Institute for Theoretical Physics, Waterloo,
 Canada, April 2004
 J.F. Traub, Toyota Institute, Chicago, IL., May 2004
 H. Wozniakowski, Monte Carlo International Conference, Juan-les-
 Pins, France, June 2004
 S. Lloyd, Newton Institute Conference on Quantum Information,
 Cambridge, UK, August 2004
 J.F. Traub, University of Rome, October 2004
 S. Lloyd and J.F. Traub, DARPA, Scottsdale, AZ, November 2004
 S. Lloyd, World Economic Forum, Davos, January 2005
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 Australia, February 2005
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 S. Lloyd, Institute for Scientific Interchange, Torino, Italy,
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 S. Lloyd, ARDA/NSA Review, Tampa, FL., August 2005
 S. Lloyd, Kodak, Rochester, NY, September 2005
 S. Lloyd, International Center for Theoretical Physics, Trieste,
 France, October 2005
 H. Wozniakowski, Bedlewo, Poland, September 2005
 S. Lloyd, Rockefeller University, NY, NY, November 2005
 J.F. Traub, University of California, Berkeley, January 2006
 J.F. Traub, Google, January 2006
 J.F. Traub, Sun, January 2006
 J.F. Traub, California Institute of Technology, March 2006
 J.F. Traub, Jet Propulsion Laboratories, March 2006
 J.F. Traub, University of California, Santa Barbara, CA, March 2006
 J.F. Traub, University of California, San Diego, CA, March 2006
 J.F. Traub, NIST, Gaithersburg, MD, April 2006

6.0 Inventions or Patent Disclosures

The President of D-Wave Company visited Columbia. The company is interested in our work on quantum algorithms for continuous problems. Columbia is applying for a patent on the Jaksch-Papageorgiou method for finding a good initial eigenvector approximation to the Abrams-Lloyd quantum algorithm for approximation of a Hermitian operator. D-Wave wishes to use this.

S. Lloyd is working with NEC to construct designs for superconducting quantum computers.

D. Cory is working with Bruker Instruments to transfer development of active feedback for improved coherent control. Progress has been made in defining the necessary electronics and showing significant improvement in performance.

S. Lloyd is working with Hewlett Packard on design and implementation of quantum controllers.

7.0 Accolades and Awards

S. Lloyd and D. Cory are members of the ARDA Quantum Computing Roadmap Committee.

S. Lloyd and J.F. Traub were interviewed by Peter Schwartz as part of the DARPA/GBN study on quantum complexity.

J.F. Traub is Editor-in-Chief, Journal of Complexity

D. Corey is Editor-in Chief, Quantum Information Processing

J.F. Traub is Chair of the Computer Science and Telecommunications Board (CSTB), National Academies

J.F. Traub was a Member, Panel of Judges, NYC Mayor's Award for Excellence in Science and Technology, 2001, 2003,2004

J.F. Traub was interviewed by SIAM. The oral interview was posted on the SIAM webpage, 2006

8.0 PhD Theses

"Exploring Large Coherent Spin Systems with Solid State NMR"

Hyung Joon Cho, MIT

"Hydrodynamic Simulations of Spin Diffusion", Daniel Greenbaum,

MIT

"Quantum Algorithms and Complexity for Certain Continuous and Related Discrete Problems", Marek Kwas, Columbia

"Achieving Reliable, Scalable, Fault-tolerant Quantum Computation", Krysta Svore, Columbia

"Quantum Chaos", Yaakov Weinstein, MIT

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